Eur. Phys. J. C 55, 309–316 (2008)
DOI 10.1140/epjc/s10052-008-0574-x

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Spontaneous Lorentz violation via QED with non-exact gauge invariance

J.L. Chkareuli[1,2,a], Z. Kepuladze[1,2], G. Tatishvili[1,2]

[1] E. Andronikashvili Institute of Physics, 6 Tamarashvili Str., 0177 Tbilisi, Georgia
[2] I. Chavchavadze State University, 32 Chavchavadze Av., 0179 Tbilisi, Georgia

Received: 13 February 2008 / Revised version: 25 February 2008 /
Published online: 27 March 2008 – © Springer-Verlag / Società Italiana di Fisica 2008

Abstract. We reconsider an alternative theory of QED with the photon as a massless vector Nambu–Goldstone boson and show that the underlying spontaneous Lorentz violation caused by the vector field vacuum expectation value, while being superficial in gauge invariant theory, becomes physically significant in QED with a tiny gauge non-invariance. This leads, through special dispersion relations appearing for charged fermions, to a new class of phenomena, which could be of distinctive observational interest in particle physics and astrophysics. They include a significant change in the GZK cutoff for UHE cosmic-ray nucleons, stability of high-energy pions and W bosons, a modification of nucleon beta decays, and some other ones.

1 Introduction

Relativistic or Lorentz invariance, while it still perfectly fits nature as we observe it, might be broken at high energies, thus manifesting itself in some new phenomena presently hidden. This has attracted considerable attention in recent years as an interesting phenomenological possibility appearing in direct Lorentz non-invariant extensions of QED and the standard model (SM) [1–9]. These extensions may in a certain measure be motivated [10, 11] by string theory, where spontaneous Lorentz violation can occur when the theory has a non-perturbative vacuum that leads to tensor-valued fields acquiring non-zero vacuum expectation values (VEVs). The VEVs are effectively a set of coupling constants, so that interactions with these coefficients have preferred spacetime directions. The full SM extension (SME) [2–7] is then defined as the effective gauge invariant field theory obtained when all such Lorentz violating VEVs are contracted term by term with SM (and gravitational) fields. However, without a completely viable string theory, it is not possible to assign definite numerical values to these coefficients. Therefore, one has in this sense a pure phenomenological approach treating the above arbitrary coefficients as quantities to be bounded in experiments as if they would simply appear due to explicit Lorentz violation. Actually, there is nothing in the SME by itself that requires that these Lorentz violation coefficients emerge due to a process of spontaneous Lorentz violation – neither the corresponding massless vector (tensor) Nambu–Goldstone (NG) bosons are required to be generated as extra physical states in the standard model, nor (especially) do these bosons have to be associated with photons or any other gauge fields of the SM.

On the other hand, however, Lorentz invariance seems to play a special role with respect to the observed internal local symmetries. The old idea [12–15] that spontaneous Lorentz invariance violation (SLIV) may lead to an alternative theory of QED, with the photon as a massless vector NG boson, still remains extremely attractive in various theoretical contexts [16][1]. In the present paper we will follow this genuine SLIV pattern causing dynamical generation of physical gauge fields, rather than the above SME approach providing a general phenomenological framework for Lorentz violation. Specifically, here we focus on the question of how this type of the SLIV (triggered by the vector field VEV), while being superficial in gauge invariant theory, may become physically significant in QED with a tiny gauge non-invariance. Notably, in contrast to the gauge invariant SME, physical Lorentz violation in the genuine SLIV model can only occur if this gauge invariance is broken. We find that such a possibility may appear when the theory is extended to include the higher dimension operators in the matter and vector fields involved. Remarkably, at the same time, the special form of physical Lorentz violation arising in the minimal dimension-five QED model happens to be one of many possible breaking patterns emerging in a general SME expansion [2–7]. This means in turn that our model is expected to be rather definite in its experimental predictions.

Before proceeding, we briefly recall some of the generic ingredients of this SLIV approach, which started long ago [12–15], in terms of models based on the four-fermion

[a] e-mail: j.chkareuli@aiphysics.ge

[1] For some recent developments, see [17–19, 21–23].

(current × current) interaction, where the Goldstonic gauge field may appear as a composite fermion–antifermion state. Unfortunately, owing to the lack of initial gauge invariance in such models and the composite nature of the NG modes that appear, it is hard to explicitly demonstrate that these modes really form together a massless vector boson as a gauge field candidate. Actually, one must make a precise tuning of the parameters, including a cancelation between terms of different orders in the $1/N$ expansion (where N is the number of fermion species involved), in order to achieve the massless photon case [15]. Rather, there are in general three separate massless NG modes, two of which may mimic the transverse photon polarizations, while the third one must be appropriately suppressed.

In this connection, the more instructive laboratory for SLIV consideration proves to be some simple class of QED type models [20–23] having from the outset a gauge invariant form, whereas spontaneous Lorentz violation is realized through the non-linear dynamical constraint

$$A^2 = n^2 M^2 \quad \left(A^2 \equiv A_\mu A^\mu, \quad n^2 \equiv n_\mu n^\mu\right) \tag{1}$$

(where n_μ is a properly oriented unit Lorentz vector, $n^2 = \pm 1$, while M is a proposed SLIV scale) imposed on the starting vector field A_μ. This constraint means in essence that the vector field A_μ develops some constant background value $\langle A_\mu(x)\rangle = (n_\mu/n^2)M$, and Lorentz symmetry $\mathrm{SO}(1,3)$ breaks down to $\mathrm{SO}(3)$ or $\mathrm{SO}(1,2)$, depending on the time-like ($n^2 > 0$) or space-like ($n^2 < 0$) SLIV. This violation provides in fact the genuine Goldstonic nature of QED, as could easily be seen from an appropriate A_μ field parametrization,

$$A_\mu = a_\mu + \frac{n_\mu}{n^2}\left(M^2 - n^2 a_\nu^2\right)^{\frac{1}{2}}, \quad n_\mu a^\mu = 0\,, \tag{2}$$

where the pure Goldstone modes a_μ are associated with the photon, while an effective Higgs mode, or the A_μ field component in the vacuum direction, is given by the square root in (2).

Actually, to appreciate the possible origin for the supplementary condition (1) one might consider the inclusion of a "standard" quartic vector field potential,

$$U(A) = -\frac{m_A^2}{2}A^2 + \frac{\lambda_A}{4}(A^2)^2 \tag{3}$$

in the conventional QED Lagrangian, as could be motivated [10, 11] to some extent by string theory. This potential inevitably causes the spontaneous violation of Lorentz symmetry in a conventional way, much as an internal symmetry violation is caused in a linear σ model for pions [24]. As a result, one has a massive Higgs mode (with mass $\sqrt{2}m_A$) together with massless Goldstone modes associated with the photon. Furthermore, just as in the pion model, one can go from the linear model for the SLIV to the non-linear one by taking the limit $\lambda_A \to \infty$, $m_A^2 \to \infty$ (while keeping the ratio m_A^2/λ_A finite). This immediately leads to the constraint (1) for the vector potential A_μ with $n^2M^2 = m_A^2/\lambda_A$, as appears from the validity of its equation of motion. Note that the correspondence with the non-linear σ model for pions may be somewhat suggestive, in view of the fact that pions are the only presently known Goldstone bosons and their theory, chiral dynamics [24], is given by the non-linearly realized chiral $\mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry rather than by an ordinary linear σ model.

The point is, however, that, in sharp contrast to the non-linear σ model for pions, the non-linear QED theory, due to the starting gauge invariance involved, ensures that all the physical Lorentz violating effects are proved to be non-observable: the SLIV condition (1) is simply reduced to a possible gauge choice for the vector field A_μ, while the S-matrix remains unaltered under such a gauge convention [20]. Really, this non-linear QED contains a plethora of particular Lorentz and CPT violating couplings when expressed in terms of the pure Goldstonic photon modes a_μ; see (2). However, contributions of these couplings to all physical processes involved are proved to be strictly canceled, as was explicitly demonstrated in the tree [20] and one-loop [21–23] approximations.

So, whereas it seems that the photon could very likely have a true Goldstonic nature, the most fundamental question of whether physical Lorentz violation takes place, which only might uniquely point toward such a possibility, is still an open question. Recall that we do not touch here on direct Lorentz non-invariant extensions of QED or the standard model when some non-covariant vector and/or matter field combinations (bilinears, trilinears etc.) are explicitly introduced in the theory [1–7]. Instead, we are looking for a pure SLIV framework, which, hand in hand with the photon appearing as a proper vector NG boson, could produce observable Lorentz violating effects.

In this context, for physical Lorentz violation to occur, an internal gauge symmetry in the theory considered should be explicitly broken rather than exact or spontaneously violated.[2] We propose that such a tiny gauge non-invariance might appear at very short distances through some higher dimension operators stemming from the gravity-influenced area. If so, physical SLIV effects would be seen in terms of powers of the ratio $M/\mathcal{M}$, where the scale $\mathcal{M}$ might be related to the Planck mass M_{P}, as would appear in some string theory scenarios, or a certain compactification scale. Notably enough, if one has such internal gauge symmetry breaking in an ordinary Lorentz invariant theory, this breaking appears to be vanishingly small at low energies, being properly suppressed by the scale $\mathcal{M}$. However, the spontaneous Lorentz violation would render it physically significant: the higher

[2] Physical SLIV effects appear to be entirely canceled in a general Abelian theory as well [23]; particularly, in the case when the internal U(1) charge symmetry is spontaneously broken hand in hand with Lorentz invariance. As a result, the massless photon being first generated by the Lorentz violation then becomes massive due to the standard Higgs mechanism, while the SLIV condition (1) in itself remains a pure gauge choice.

the scale M, the greater the SLIV effects observed. Remarkably, the gauge non-invariance proposed cannot generate the photon mass since photons appear in the theory as vector NG bosons and, therefore, their masslessness is guaranteed by the SLIV. The absence of longitudinal photons in the theory together with strict conservation of the Noether fermion current involved provides, on the other hand, conservation of electric charge as well.[3]

To put all this another way, note that gauge invariance in Goldstonic QED appears in essence as a necessary condition for the starting vector field A_μ not to be superfluously restricted in degrees of freedom, apart from the SLIV constraint (1) due to which the true vacuum in the theory is chosen [26, 27]. For any extra restriction(s) imposed on the vector field, it would be impossible to set the required initial conditions in the appropriate Cauchy problem and, in quantum theory, to choose self-consistent equal-time commutation relations [28]. From this point of view, the only possible theory compatible with the SLIV condition (1) appears to be just conventional gauge invariant QED. One may expect, however, that quantum gravity could in general hinder the setting of the required initial conditions at extra-small distances, thus admitting a superfluous restriction of the starting vector field A_μ. This, eventually, through some high order operators, would manifest itself in a violation of the above gauge invariance, which in turn might bring the spontaneous Lorentz violation to low energies. We suggest here such a type of model (Sect. 2) and explore some of its immediate physical and astrophysical consequences (Sect. 3). Our conclusions are given in Sect. 4.

2 The model

Before proceeding to the extension of conventional QED to the higher dimension couplings included, we are reminded that gauge invariance in standard quantum electrodynamics is not necessarily postulated for the photon–fermion interaction – it appears on its own if, apart from relativistic invariance, the restrictions related with the conservation of parity, charge-conjugation symmetry and number of fermions are also imposed in the Lagrangian. Actually, one uses gauge invariance only if one constructs the photon kinetic term to have the ordinary form $F_{\mu\nu}F^{\mu\nu}$, since this is necessary in order that the Hamiltonian be bounded below.[4] Similarly, analogous restrictions for photon–fermion couplings of higher dimensions generally allow only for a few new ones (for each order in the theory's inverse scale $1/\mathcal{M}$), which appear to possess, however, some approximate gauge invariance rather than an exact one as one has in conventional QED with dimensionless coupling constants. In this connection the most transparent situation arises in the minimal QED extension to dimension-five couplings, which we consider here in detail. Since this extension, apart from photon–fermion interaction terms, will necessarily include the free fermion bilinear of type $(1/\mathcal{M})\partial_\mu\overline{\psi}\partial^\mu\psi$, one could have the idea that free fermions could generally be described by some combined Dirac–Klein–Gordon equation rather than the pure Dirac equation, what might be hidden at low energies. However, due to spontaneous Lorentz violation this "fermion–boson complementarity" could become significant, providing a somewhat natural model for a tiny gauge non-invariance in QED when the electromagnetic interaction is "switched on". As a result, the SLIV, having been superficial in gauge invariant theory, becomes in fact physically observable through a certain dispersion relation, which automatically appears for charged fermions. This is in contrast to the direct Lorentz violation models [2–9], where some modified dispersion relations for the photon and/or matter particles involved are in essence specially postulated.

One can start with a free Lagrangian for some massive charged fermion ψ in the form

$$L(\psi) = \overline{\psi}\big(\mathrm{i}\gamma_\mu\partial^\mu - m_0\big)\psi + \frac{1}{\mathcal{M}}\partial_\mu\overline{\psi}\cdot\partial^\mu\psi\,, \tag{4}$$

which contains, apart from a true fermionic kinetic term, some "bosonic" type kinetic term as well. As is clear from this Lagrangian, the fermion dispersion relation will be changed a little, so that for its four-momentum p_μ squared one has

$$p^2 = \big(m_0 - p^2/\mathcal{M}\big)^2 = m_0^2(1 - 2m_0/\mathcal{M} + \cdots)\,, \tag{5}$$

which leads to a tiny mass shift for the fermion, which, of course, is of no experimental interest. Let us now turn on all possible interaction terms, which, under the foregoing discrete and global symmetry restrictions taken, amount to the gauge type "minimal" interactions of the fermion with the vector field (given by the standard replacement $\partial^\mu \to \partial^\mu + \mathrm{i}eA^\mu$) through both of the kinetic terms involved in the Lagrangian L; see (4). In this connection, there might appear the question of whether the "fermionic" and "bosonic" type couplings of the ψ field with the vector field A^μ have the same coupling constant e. If so, the total Lagrangian with the above "minimal" interaction included, while being non-renormalizable, will still be left gauge invariant. However, generally, these coupling constants are different, which means that the Lagrangian is no more gauge invariant as soon as one takes into account the small "bosonic" type kinetic term in (4) being suppressed by the scale $\mathcal{M}$. This is just a type of gauge non-invariance that underlies our model leading eventually to physical Lorentz violation. So, the initially Lorentz invariant theory for fermion–vector field interactions, which possesses a slightly broken gauge invariance, is given by the general

[3] Note, at the same time, that if electric charge non-conservation would occur, this violation would appear, as we see in Sect. 2, well below the presently existing bounds [25].

[4] Note also that a general photon kinetic term gives rise to ghosts in the propagator and, specifically in the Lorentz violating QED type theory, to a domain wall solution for the vector potential A_μ that might lead to a wall-dominated early Universe and its immediate collapse [29].

Lagrangian[5]

$$L(A,\psi) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\left[\mathrm{i}\gamma_\mu D^\mu - m_0\right]\psi + \frac{1}{\mathcal{M}}D'^{*}_{\mu}\overline{\psi}\cdot D'^{\mu}\psi\,, \quad (6)$$

containing, apart from the "true fermionic" terms with covariant derivative $D^\mu = \partial^\mu + \mathrm{i}eA^\mu$, the "bosonic" type terms as well, with $D'^\mu = \partial^\mu + \mathrm{i}e'A^\mu$, either taken with independent charges e and e', respectively. Remarkably, despite the fact that both the "fermionic" and "bosonic" parts of the Lagrangian (6) are individually gauge invariant, gauge invariance is in fact broken when they are taken together. As a result, though this Lagrangian practically (i.e. neglecting the last term in (6)) does not differ from the conventional QED Lagrangian, provided that the vector field A_μ is associated with a photon, a drastic difference appears when this field develops a VEV and the SLIV occurs.

Actually, putting the SLIV parameterization (2) into our basic Lagrangian (6) one comes to a truly Goldstonic model for the QED. This model contains, among other terms, the inappropriately large (while false) Lorentz violating fermion bilinear $-eM\overline{\psi}(n_\mu\gamma^\mu/n^2)\psi$, which appears when the effective Higgs field expansion (as is given in the parametrization (2)) in the true Goldstone modes a_μ is applied to the fermion current interaction term $-\overline{\psi}\gamma_\mu A^\mu\psi$ in the "fermionic" part of the Lagrangian $L(A,\psi)$. However, due to local invariance of this part, this bilinear term can be gauged away by making an appropriate redefinition of the fermion field $\psi \to \mathrm{e}^{-\mathrm{i}e\omega(x)}\psi$ with a gauge function $\omega(x)$ linear in the coordinates, $\omega(x) = (n_\mu x^\mu/n^2)M$. Meanwhile, the small "bosonic" part being gauge non-invariant is appropriately changed under this redefinition. So, one eventually arrives at the essentially non-linear SLIV Lagrangian for the photon–fermion interaction with the significantly modified fermion bilinear terms

$$\mathcal{L}(a_\mu,\psi) = L\big(A_\mu \to a_\mu + n_\mu(a^2/2M+\cdots),\psi\big) - \mathrm{i}\Delta e\frac{M}{\mathcal{M}}\frac{n_\mu}{n^2}\overline{\psi}\overleftrightarrow{\partial^\mu}\psi + (\Delta e)^2 n^2\frac{M^2}{\mathcal{M}}\overline{\psi}\psi\,, \quad (7)$$

where we have explicitly indicated that the vector field A_μ in the starting Lagrangian L (6) is replaced by the pure Goldstone field a_μ associated with the photon (appearing in the gauge $n_\mu a^\mu = 0$) plus the effective Higgs field expansion in (2). We also retained the notation ψ for the redefined fermion field and denoted, as usually, $\overline{\psi}\overleftrightarrow{\partial^\mu}\psi = \overline{\psi}(\partial^\mu\psi) - (\partial^\mu\overline{\psi})\psi$. Note that the extra fermion bilinear terms[6] given in the second line in (7) are produced just due to the gauge invariance breaking that is determined by the electromagnetic charge difference $\Delta e = e' - e$ in the starting Lagrangian L; see (6). As a result, there appears the entirely new, SLIV inspired, dispersion relation for a charged fermion (taken with four-momentum p_μ) of the type

$$p_\mu^2 \cong \left[m + 2\delta\big(p_\mu n^\mu/n^2\big)\right]^2\,, \quad m = m_0 - \delta^2 n^2\mathcal{M}\,, \quad (8)$$

given to an accuracy of $O(m^2/\mathcal{M}^2)$. Here δ stands for the small characteristic, positive or negative, parameter $\delta = (\Delta e)M/\mathcal{M}$ of the physical Lorentz violation that reflects the joint effect as given, from the one hand, by the SLIV scale M, and, from the other, by the charge difference Δe being a measure of the internal gauge non-invariance. Notably, space-time by itself still possesses Lorentz invariance; however, fermions with the SLIV contributing into their total mass $m = m_0 - \delta^2 n^2\mathcal{M}$ propagate and interact in it in a Lorentz non-covariant way.[7] At the same time, the photon dispersion relation is retained in the order $1/\mathcal{M}$ considered.[8]

Let us now try to estimate the possible scale of Lorentz violation and a numerical value of the parameter δ, being in essence the only measure of the physical Lorentz violation in our model. Some estimation could follow from the naturalness requirement that the free fermion mass presented in the Goldstonic QED Lagrangian (7), and, specifically the mass of the lightest charged fermion, which is the electron mass, should not be significantly disturbed by the Lorentz violation. Otherwise possible fine tuning between the SLIV contribution to this mass and its starting value would become necessary. Proposing the SLIV contributed total electron mass m_e to remain of the same order as the starting mass m_{0e}, one comes from (8) to the inequality $\delta^2\mathcal{M} \lesssim m_e$. Remarkably, the characteristic parameter δ depends on neither the SLIV scale M nor on the charge difference Δe individually, but on their product only, and, for the above "stability condition" against the SLIV contribution of the electron mass, it is generally given by the range of values

$$\delta = (\Delta e)M/\mathcal{M}\,, \quad |\delta| \lesssim \overline{\delta} \equiv \sqrt{m_e/\mathcal{M}}\,, \quad (9)$$

[5] For the sake of simplicity we have not included into the Lagrangian $L(A,\psi)$ the anomalous magnetic moment type coupling $\frac{e''}{\mathcal{M}}F^{\mu\nu}\overline{\psi}\sigma_{\mu\nu}\psi$, which is gauge invariant on its own and appears inessential for what follows. Another simplification is that we have omitted an independent "sea-gull" type coupling $\frac{e'''}{\mathcal{M}}A_\mu^2\overline{\psi}\psi$ in the Lagrangian (a term like that is already contained in its "bosonic" part), since such a coupling due to the SLIV condition (1) is simply reduced to some inessential correction to the fermion mass term. All things considered, the Lagrangian $L(A,\psi)$ gives in fact the most general extension of QED in $\frac{1}{\mathcal{M}}$ order, taken under the Lorentz and extra discrete and global symmetry restrictions discussed above.

[6] Notably, from a general SME point of view one could say that just this form of physical Lorentz violation known as "e-term" breaking [2–7] appears to dominate, due to the genuine SLIV pattern considered, over many other Lorentz breaking terms emerging in the SME.

[7] Note also that the fermion dispersion relation (8) is substantially different from the dispersion relations extensively used before [8, 9] where just the mass squared happened to be shifted in the preferred frame rather than the mass by itself as in (8).

[8] One must, of course, expect that non-gauge invariant photon kinetic terms, changing its dispersion relation, are also generated through radiative corrections. But these terms are down by high orders in $1/\mathcal{M}$ relative to the basic $F_{\mu\nu}^2$ term taken, and, therefore, can be neglected.

which for a sufficiently high mass scale $\mathcal{M}$ happens by itself (as we see below) to be of a certain interest for current high-energy tests of special relativity. Particularly, when taking just the Planck mass M_{P} for the highest scale in the theory ($\mathcal{M} = M_{\mathrm{P}}$), one has the upper limits

$$|\delta| \lesssim \overline{\delta} = 6.5 \times 10^{-12}\,, \quad M \lesssim 10^{8}(e/\Delta e)\,\mathrm{GeV} \tag{10}$$

for the δ parameter and the Lorentz violation scale M, respectively.

Before proceeding to applications, let us note that, in the order $1/\mathcal{M}$ considered, all other particles apart from charged fermions, such as photon, neutrinos, weak bosons etc. are proposed to satisfy the standard dispersion relations. Inclusion of new charged fermions into the Goldstonic QED Lagrangian (7) will in general increase the number of the SLIV parameters in the theory by assigning to every fermion species f (being some lepton or quark) its own δ_f parameter. These parameters, as is seen from (9), will actually differ from one another by the corresponding charge differences $(\Delta e)_f$ only. This immediately leads to the conclusion that the δ parameters for particles and antiparticles must be equal but of opposite sign. Apart from that, some of the charge differences might appear to be equal if certain symmetries for leptons and quarks are postulated; say, grand unified symmetry inside a lepton–quark family and/or flavor symmetry between families.

3 Some immediate applications

One may now see that, due to the spontaneous Lorentz violation resulting in the new dispersion relation (8) for charged fermions, the kinematics of processes in which such fermions are participating is substantially changed. At low energies these changes can be neglected, but at high energies they may play a crucial role. As a result, some of the allowed processes appear to be suppressed at high energies and, on the contrary, some of the suppressed processes are now allowed to go. This could substantially change the particle phenomenology at high energies, which would lead to some new observations, as well as corrections to the early Universe scenario. Certain of these processes were previously discussed in direct Lorentz violation scenarios [2–9]. Predictions of our SLIV model appear in fact to be more distinctive being dependent on only a few SLIV parameters δ (9) assigned to elementary charged fermions, quarks and leptons. Actually, all changes as compared with conventional QED can readily be derived replacing the masses m_f of these fermions by their non-covariant "effective" masses:

$$m_f^{*} \equiv \sqrt{p_\mu^2} \cong |m_f + 2\boldsymbol{\delta}_f p_0|\,, \tag{11}$$

as follows from the above dispersion relation (8), where we also introduced a modified (two-component) parameter $\boldsymbol{\delta}_f$, which is equal to $\boldsymbol{\delta}_f = \delta_f$ for the time-like SLIV and $\boldsymbol{\delta}_f = \delta_f \cos\theta$ for the space-like SLIV, respectively. Note that in the high-energy region that we are interested in, the scalar product $p_\mu n^\mu/n^2$ in (8) for the space-like SLIV ($n^2 < 0$) with the angle θ between the fermion three-momentum $\vec{p}$ and the Lorentz violation direction vector $\vec{n}$ just reduces to $p_\mu n^\mu/n^2 = |\vec{p}|\cos\theta \cong p_0 \cos\theta$.

Consideration of composite hadron states, mesons and baryons, in our model needs further clarification. Generally, one could assign to each of these composites its own $\boldsymbol{\delta}$ parameter, or its own effective mass m^* (11), which would lead to a plethora of new SLIV parameters in the model. However, we propose the following simple rules for composites that might naturally work. Actually, one may treat SLIV features of hadrons solely based on their quark content so that their effective masses are additively combined with those of the quarks and antiquarks involved, both taken at the same energy E in a preferred frame. So, for some meson φ consisting of quark q_1 and antiquark $\overline{q}_2$, this effective mass might look like

$$m_\varphi^{*} = |m_\varphi + 2(\boldsymbol{\delta}_1 - \boldsymbol{\delta}_2)E|\,, \tag{12}$$

where we have used that the $\boldsymbol{\delta}$ parameter for an antiquark has an opposite sign ($\boldsymbol{\delta}_{q_1} \equiv \boldsymbol{\delta}_1$, $\boldsymbol{\delta}_{\overline{q}_2} = -\boldsymbol{\delta}_{q_2} \equiv -\boldsymbol{\delta}_2$), as was indicated at the end of Sect. 2, and we replaced the sum of the current quark masses $m_1 + m_2$ in (12) by the meson invariant mass m_φ. This replacement seems to be a quite good approximation for mesons consisting of heavy c, b and t quarks, but not for mesons consisting of light quarks u, d and s, whose current masses $m_{u,d,s}$ hardly provide masses of the corresponding mesons (and baryons). The point is, however, that the color interaction converting these current quark masses into the constituent quark ones (and leading eventually to the physical hadron masses) is presumably Lorentz invariant, so that the non-covariant part in the meson effective mass (12) with the $\boldsymbol{\delta}$ parameters depending solely on the quark electric charge differences $(\Delta e)_{q_{1,2}}$ seems to be basically preserved. Analogously, the dispersion relations of the baryons are always frame-dependent, being determined by the particular quark content in its effective mass:

$$m_B^{*} = |m_B + 2(\boldsymbol{\delta}_1 + \boldsymbol{\delta}_2 + \boldsymbol{\delta}_3)E|\,, \tag{13}$$

provided that baryon B with invariant mass m_B is composed of the quarks q_1, q_2 and q_3 with parameters $\boldsymbol{\delta}_{1,2,3}$.

A few simple remarks are in order. As is readily seen from (12), mesons that are diagonal in the quark flavors, such as π^0, η, ρ^0, ϕ, J/Ψ etc., have zero $\boldsymbol{\delta}$ parameters and thus they obey the standard dispersion relations. Furthermore, mesons and baryons with the same quark content possess equal $\boldsymbol{\delta}$ parameters and, therefore, have alike effective masses. And, in a similar manner as in the elementary fermion case, the $\boldsymbol{\delta}$ parameters for composite hadrons and their antiparticles appear to be equal but of opposite sign.

3.1 GZK cutoff revised

One of the most interesting examples where a departure from Lorentz invariance can essentially affect a physical process is the transition $p + \gamma \rightarrow \Delta$, which underlies the Greisen–Zatsepin–Kouzmin (GZK) cutoff for ultra-high energy (UHE) cosmic rays [30, 31]. According to this idea primary high-energy nucleons (p) should suffer an inelastic impact with cosmic background photons (γ) due to the resonant formation of the first pion–nucleon resonance $\Delta(1232)$, so that nucleons with energies above $\sim 5\times10^{19}$ eV could not reach us from further away than ~ 50 Mpc. During the last decade there were serious indications [32–34] that the primary cosmic-ray spectrum extends well beyond the GZK cutoff, though presently the situation is somewhat unclear due to a certain criticism of these results and new data that recently appeared [35]. However, no matter how things will develop, we could say that according to the new fermion dispersion relation (8) the GZK cutoff will necessarily be changed (increased or decreased, depending on the sign of the corresponding δ parameter) at superhigh energies. Remarkably, for the Planck mass scale case in the theory ($\mathcal{M} = M_{\mathrm{P}}$) the above transition, providing this cutoff, appears to be significantly weakened (or even completely undone) just around the aforementioned GZK energy region, as one can see from the δ parameter value range (10) calculated for this case.

Really, we must replace the fermion masses in a conventional proton threshold energy for this process by their effective masses $m^*_{p,\Delta} \cong |m_{p,\Delta} + 2\boldsymbol{\delta} E_{p,\Delta}|$, which can be taken with equal $\boldsymbol{\delta}$ parameters as for composite states having a similar quark content ($\boldsymbol{\delta} \equiv \boldsymbol{\delta}_p = \boldsymbol{\delta}_\Delta = 2\boldsymbol{\delta}_u + \boldsymbol{\delta}_d$). Using then the approximate equality of their energies, $E_\Delta = E_p + \omega \cong E_p$, since the target photon energies ω are vanishingly small (being a thermal distribution with temperature $T = 2.73$ K, or $kT \equiv \overline{\omega} = 2.35\times10^{-4}$ eV), one comes to the condition determining the proton energy region in which the foregoing transition is kinematically forbidden for a head-on impact:

$$E_p > \frac{m^2_\Delta - m^2_p}{4[\omega - \boldsymbol{\delta}(m_\Delta - m_p)]} = \frac{6.8}{\omega/\overline{\omega} - 8.1\boldsymbol{\delta}/\overline{\delta}} \times 10^{20}\,\mathrm{eV}\,. \tag{14}$$

As one can readily see, the SLIV modification of the proton threshold energy E_p in (14) might naturally relax the GZK cutoff and even permits UHE cosmic-ray nucleons to travel cosmological distances (when $\omega/\overline{\omega} \approx 8.1\boldsymbol{\delta}/\overline{\delta}$ with $\overline{\delta}$ given in (10)) provided that the $\boldsymbol{\delta}$ parameter in (14) is taken positive. Conversely, for negative values the original GZK cutoff tends to a decrease. Most interestingly, there is predicted some marked spatial anisotropy for primary nucleons in the space-like Lorentz violation case ($\boldsymbol{\delta} = \delta\cos\theta$), which results in an ordinary GZK cutoff for perpendicular (to the SLIV vector $\vec{n}$) direction, whereas it is lower or higher for other directions.

3.2 Stability of high-energy vector and scalar bosons

Another interesting example is provided by the decays of vector and scalar bosons into fermions, no matter whether they all are elementary or composite. Usually these processes are possible if the boson mass $\mathfrak{m}$ is no less than the sum of the fermion invariant masses $m_{1,2}$, but now, when fermions and (composite) bosons can have some effective masses given by (11)–(13), these decays at high energies may appear to be kinematically suppressed, as can easily be confirmed. Actually, for a particular two-body decay case this process appears to be banned if the inequality $\mathfrak{m}^* < m^*_1 + m^*_2$ for the fermion effective masses $m^*_{1,2}$ is satisfied for the minimum total energy of decay products with a given total momentum $\vec{P}$. It follows that all momenta are collinear in the configuration of the minimum total energy, and the fermion momenta are equal to

$$\vec{p}_{1,2} = \frac{m_{1,2}}{m_1 + m_2}\vec{P}\,, \tag{15}$$

so that at energies

$$E > \frac{1}{2}(\mathfrak{m} - m_1 - m_2)\frac{m_1 + m_2}{\boldsymbol{\delta}_1 m_1 + m_2\boldsymbol{\delta}_2 - \boldsymbol{\delta}(m_1 + m_2)} \tag{16}$$

this boson could appear stable.

Applying this result to the weak W boson decays into quarks and leptons ($\mathfrak{m} = m_W$, $\boldsymbol{\delta} \equiv \boldsymbol{\delta}_W = 0$)[9] and taking the $\boldsymbol{\delta}_{1,2}$ parameters to be of the same order as those that are required for a weakened GZK cutoff version ($\boldsymbol{\delta}_{1,2} \sim \boldsymbol{\delta}_{p,\Delta} \sim 10^{-12}$), we find that stable W bosons appear at the energy region $\sim 10^{23}$ eV that seems to be somewhat problematic to be directly detected. At the same time, the Z and Higgs bosons, which are only related to the flavor-diagonal quark and lepton currents, do not change their decay rates with energy since, as already noted, the SLIV effects from particles and antiparticles are expected to be canceled.

However, the special observational interest may cause charged pion stability at high energies against the standard $\pi \to \mu + \nu$ decays. In contrast to the W boson, the composite charged pion has a non-zero SLIV parameter $\boldsymbol{\delta}_\pi = \boldsymbol{\delta}_u - \boldsymbol{\delta}_d$ (expressed in the up and down quark parameters $\boldsymbol{\delta}_{u,d}$; see (12)) and, therefore, the non-covariant effective mass is $m^*_\pi = |m_\pi + 2\boldsymbol{\delta}_\pi E_\pi|$. So, properly adjusting the general formula (16) for the two-body decays ($\mathfrak{m} = m_\pi$, $\boldsymbol{\delta} = \boldsymbol{\delta}_\pi$; $m_1 = m_\mu$, $\boldsymbol{\delta}_1 = \boldsymbol{\delta}_\mu$; $m_2 = m_\nu = 0$, $\boldsymbol{\delta}_2 = \boldsymbol{\delta}_\nu = 0$), one eventually has for the threshold energy providing pion stability ($m^*_\pi < m^*_\mu$)

$$E_\pi > \frac{1}{2}\frac{m_\pi - m_\mu}{\boldsymbol{\delta}_\mu - \boldsymbol{\delta}_\pi}\,. \tag{17}$$

This energy region, when the muon and pion $\boldsymbol{\delta}$ parameter values ($\boldsymbol{\delta}_\mu - \boldsymbol{\delta}_\pi > 0$) are taken to be of the same order $\sim 10^{-12}$ as in the foregoing cases, appears to be significantly lower than that for the stable W boson, being just near the GZK cutoff energy $\sim 10^{19}$ eV. Thus, the UHE primary cosmic rays may include stable charged pions that could in principle be detected at current experiments [35], whereas neutral pions being diagonal quark–antiquark composites are left to be very unstable, as they usually are. Again, for the space-like SLIV case spatial anisotropy is

[9] For the pure leptonic decay $W \to l\overline{\nu}$, (16) is maximally simplified, $E_B > (m_W - m_l)/2\boldsymbol{\delta}_l$, since the neutrino is presumably massless and has a normal dispersion relation ($m_2 = 0$ and $\delta_2 = 0$).

expected, according to which the stable charged pions are predicted to be largely located along the SLIV direction $\vec{n}$.

3.3 Modified nucleon decays

As a last example we consider ordinary neutron β decay ($n \to pe^-\overline{\nu}$). Since the neutron is heavier than the proton, $m_n > m_p$, usually neutron β decay is allowed, while proton β decay ($p \to ne^+\nu$) is kinematically suppressed. However, due to the Lorentz non-invariance their effective masses may grow at high energies in such a way that $m_n^* < m_p^*$ in a preferred frame. This means that neutron and proton change places – the neutron becomes stable, whereas the proton decays. Using the above general formula (16), one can readily find the threshold energy value when this happens,

$$E > \frac{m_n - m_p}{2(\boldsymbol{\delta}_p - \boldsymbol{\delta}_n)} = \frac{m_n - m_p}{2(\boldsymbol{\delta}_u - \boldsymbol{\delta}_d)}\,, \tag{18}$$

where we have treated both beta processes as essentially two-body decays with lepton masses ignored. Again with the $\boldsymbol{\delta}_{p,n}$ parameters $\sim 10^{-12}$ taken as in the foregoing examples, one finds the energy region $E > 10^{18}$ eV, which is an active research area for current cosmic-ray experiments [30–34]. At these energies stable neutrons, it follows, can be contained in primary UHE cosmic rays, whereas unstable protons cannot.

To conclude, we have considered some basic applications of the model that can be described in terms of a few $\boldsymbol{\delta}$ parameters assigned to the elementary fermions, quarks and leptons. Our Lorentz violating predictions appear to be quite certain for the above processes, being conditioned just by the vector field model of the SLIV. At the same time this minimal model predicts the strictly vanishing effects in many processes (where generally some Lorentz violation might in principle be expected), such as the Z and Higgs boson and photon decays, decays of diagonal quark–antiquark composites (π^0, η, ρ^0, ϕ, J/Ψ etc.), neutrino oscillations and others, which have been previously discussed on pure hypothetical grounds [2–9].

4 Summary and outlook

We have argued that genuine spontaneous Lorentz violation, which in the QED framework would induce the photon as a vector NG boson, does not manifest itself in any physical way due to the gauge invariance involved. In essence, the SLIV ansatz taken as $A_\mu(x) = a_\mu(x) + n_\mu M$ may be treated by itself as a pure gauge transformation with a gauge function linear in the coordinates, $\omega(x) = (n_\mu x^\mu)M$. In this sense, gauge invariance in QED leads to the conversion of the SLIV into gauge degrees of freedom of the massless photon. This is what one could refer to as the generic non-observability of the SLIV in QED. Furthermore, as was shown some time ago [36, 37], gauge theories, both abelian and non-abelian, can be obtained by themselves from the requirement of the physical non-observability of the SLIV, caused by the Goldstonic nature of the vector fields, rather than from the standard gauge principle.

All this requires that gauge invariance in QED should be broken rather than be exact. We have proposed some simple model for a tiny gauge non-invariance that might be caused by quantum gravity at extra-small distances through some higher order operators involved. To this end we extended QED to the lowest order in $1/\mathcal{M}$ (the theory's inverse scale) so that all possible dimension-five operators compatible with the accompanying global and discrete symmetries are included. They appear to possess in general some approximate gauge invariance rather than an exact one as in conventional QED with dimensionless coupling constants. This in turn leads of necessity to the physical Lorentz violation resulting in a new dispersion relation (8) for charged fermions. As a consequence, kinematics of processes in which such fermions are participating is substantially changed. While at low energies these changes can be neglected, at high energies they may play a crucial role, as was illustrated by the examples of the processes given above.

We have so far considered the theory of QED by itself. However, this theory should be included into the standard model with its internal gauge symmetry $SU(2) \times U(1)_Y$ to have a fully realistic framework. This will substantially change the entire approach, though the physical consequences are largely preserved and, moreover, extended. Note first of all that the SLIV condition (1) is taken now for the $U(1)_Y$ hypercharge gauge field B_μ rather than for the electromagnetic one, A_μ, which by itself appears later when the starting $SU(2) \times U(1)_Y$ symmetry is spontaneously broken. Furthermore, for the physical Lorentz violation to occur, this $U(1)_Y$ hypercharge gauge invariance should be explicitly broken by some high dimension operators, supposedly induced by quantum gravity. It is apparent, on the other hand, that the chiral nature of this symmetry in the SM forbids quarks and leptons to form the "bosonic" type terms, which in the pure QED framework were taken for a vector-like fermion in the Lagrangians (4) and (6). This means that there cannot appear $U(1)_Y$ breaking terms in $1/\mathcal{M}$ order, as happened in the above QED case. However, they appear, as one can readily see, in the next order, $1/\mathcal{M}^2$. Using again the restriction requirements related to all discrete and global symmetries involved one can reduce the number of possible breaking terms to a few ones including some "gravity type" coupling,

$$\frac{1}{\mathcal{M}^2} B_\mu B_\nu \Theta^{\mu\nu}\,, \tag{19}$$

[10] Among possible couplings the foregoing breaking terms emerging in the pure QED Lagrangian (6) could of course be induced in the SM as well. They arise when the dimension-six coupling $(\phi/\mathcal{M}^2)D'^*_\mu\overline{\psi}_{\mathrm{L}}D'^\mu\psi_{\mathrm{R}}$ appears and the conventional Higgs doublet ϕ acquires its VEV. However, these terms are down by the tiny ratio of the electroweak scale to the SLIV one as compared with the chirality preserving couplings.

[11] These couplings are like the potential terms (3) introduced in the QED case – they in a similar manner will lead to the SLIV condition $B^2 = n^2M^2$.

where $\Theta^{\mu\nu}$ stands for a total energy-momentum tensor of all fermion and vector fields involved, which is proposed to be symmetrical, conserving and gauge invariant.[10] Remarkably enough, the coupling (19) appears as the only possible one if one further requires that these breaking terms possess a partial $\mathrm{U}(1)_Y$ gauge symmetry in the sense that, while the theory is basically $\mathrm{U}(1)_Y$ gauge invariant (being constructed from ordinary covariant derivatives of all matter fields involved), the B_μ field by itself is allowed to form its own polynomial couplings[11], and it also may appear as factors in other field couplings. Such couplings, and specifically the coupling (19), cause a tiny $U(1)_Y$ gauge non-invariance in the SM and lead eventually to the physical Lorentz violation. As a result, after a spontaneous $\mathrm{SU}(2)\times\mathrm{U}(1)_Y$ symmetry breakdown, one comes to the changed dispersion relation for all quarks and leptons including neutrinos, since they all possess hypercharge and thus are related to the vector field B_μ developing a VEV. Another peculiarity related to an extension to the SM is that the gauge non-invariance now appears not only in the fermion sector of theory, but also in the vector field sector in itself. So, one eventually comes to the changed dispersion relation for the photon as well that leads in turn to extra interesting manifestations to be observed. The entire framework for a study of spontaneous Lorentz violation in the standard model with non-exact gauge invariance we plan to consider in detail elsewhere.

Acknowledgements. One of us (J.L.C.) thanks J. Bjorken, C. Froggatt, R. Jackiw, R. Mohapatra and H. Nielsen for interesting correspondence, useful discussions and comments. Financial support from World Federation of Scientists is gratefully acknowledged by J.L.C. and Z.K.

References

1. S.M. Carroll, G.B. Field, R. Jackiw, Phys. Rev. D **41**, 1231 (1990)
2. D. Colladay, V.A. Kostelecky, Phys. Rev. D **55**, 6760 (1997)
3. D. Colladay, V.A. Kostelecky, Phys. Rev. D **58**, 116 002 (1998)
4. V.A. Kostelecky, R. Lehnert, Phys. Rev. D **63**, 065 008 (2001)
5. V.A. Kostelecky, Phys. Rev. D **69**, 105 009 (2004)
6. R. Bluhm, V.A. Kostelecky, Phys. Rev. D **71**, 065 008 (2005)
7. V.A. Kostelecky (ed.), CPT and Lorentz Symmetry (World Scientific, Singapore, 1999, 2002, 2005)
8. S. Coleman, S.L. Glashow, Phys. Lett. B **405**, 249 (1997)
9. S. Coleman, S.L. Glashow, Phys. Rev. D **59**, 116 008 (1999)
10. V.A. Kostelecky, S. Samuel, Phys. Rev. D **39**, 683 (1989)
11. V.A. Kostelecky, R. Potting, Nucl. Phys. B **359**, 545 (1991)
12. J.D. Bjorken, Ann. Phys. (New York) **24**, 174 (1963)
13. T. Eguchi, Phys. Rev. D **14**, 2755 (1976)
14. H. Terazava, Y. Chikashige, K. Akama, Phys. Rev. D **15**, 480 (1977)
15. M. Suzuki, Phys. Rev. D **37**, 210 (1988)
16. C.D. Froggatt, H.B. Nielsen, Origin of Symmetries (World Scientific, Singapore, 1991)
17. J.D. Bjorken, hep-th/0111196
18. P. Kraus, E.T. Tomboulis, Phys. Rev. D **66**, 045 015 (2002)
19. A. Jenkins, Phys. Rev. D **69**, 105 007 (2004)
20. Y. Nambu, Prog. Theor. Phys. Suppl. Extra, 190 (1968)
21. J.L. Chkareuli, C.D. Froggatt, R.N. Mohapatra, H.B. Nielsen, hep-th/0412225
22. A.T. Azatov, J.L. Chkareuli, Phys. Rev. D **73**, 065 026 (2006)
23. J.L. Chkareuli, Z.R. Kepuladze, Phys. Lett. B **644**, 212 (2007)
24. S. Weinberg, The Quantum Theory of Fields, vol. 2 (Cambridge University Press, Cambridge, 2000)
25. Particle Data Group, S. Eidelman et al., Phys. Lett. B **592**, 1 (2004)
26. J.L. Chkareuli, C.D. Froggatt, J.G. Jejelava, H.B. Nielsen, Nucl. Phys. B **796**, 211 (2008) arXiv:0710.3479 [hep-th]
27. J.L. Chkareuli, J.G. Jejelava, Phys. Lett. B **659**, 754 (2008)
28. V.I. Ogievetsky, I.V. Polubarinov, Ann. Phys. (New York) **25**, 358 (1963)
29. J.L. Chkareuli, C.D. Froggatt, H.B. Nielsen, hep-th/0610186
30. K. Greizen, Phys. Rev. Lett. **16**, 748 (1966)
31. G.T. Zatsepin, V.A. Kuz'min, JETP Lett. **41**, 78 (1966)
32. Fly's Eye Collaboration, D.J. Bird et al., Astrophys. J. **424**, 144 (1995)
33. AGASA Collaboration, M. Takeda et al., Phys. Rev. Lett. **81**, 1163 (1998)
34. AGASA Collaboration, M. Takeda et al., Astropart. Phys. **19**, 447 (2003)
35. Pierre Auger Collaboration, J. Abraham et al., arXiv: 0712.1147 [astro-ph]
36. J.L. Chkareuli, C.D. Froggatt, H.B. Nielsen, Phys. Rev. Lett. **87**, 091 601 (2001)
37. J.L. Chkareuli, C.D. Froggatt, H.B. Nielsen, Nucl. Phys. B **609**, 46 (2001)